HEAT AND MASS TRANSPORT IN BUNDLES THAT MOVE AND GIVE OFF GAS

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The formation of fibers from polymer melts is one of the basic ways to produce synthetic threads. Modeling these processes mathematically makes it possible to expose the basic physical factors which affect the formation, to find the most appropriate operating conditions and methods, and to estimate the parameters of the basic machinery. Construction of complete mathematical models of such processes is still far from finished. Existing calculational methods make it possible to find distribution parameters for a single thread [1]. However, in actual industrial processes, tens or hundreds of elementary fibers are drawn simultaneously. A method has been suggested and tested [2, 3] to model the motion and heat transport in fiber bundles. It allows flow parameters to be calculated and can serve as a basis for constructing more exact models for drawing combined fibers in an open fluid and in various devices.

However, in some methods of fiber formation ["wet" and "dry"] and in many processes for producing fiber bundles, determining how the fibers interact with each other and with the external fluid is affected by mass-transport processes. Thus, in making thermally stable carbon fibers, the polymer (matrix) is burned off, and gaseous combustion products are given off. The drying and and the "dry" formation of combined fibers are accompanied by the evaporation of the liquid, and "wet" formation occurs due to mass transport between the fiber and the fluid. Formation from a melt can be accompanied by partial decomposition of the polymer and the evolution of toxic low molecular weight compounds. Removing these compounds and neutralizing them is a serious ecological problem. All these examples indicate the urgency for constructing models of heat and mass transport in fiber bundles from whose surfaces additional gas and liquid are given off. Therefore there is interest in expanding previously derived methods for calculating heat transport in rod bundles and in applying them to modeling heat and mass transport in fiber systems that give off gas.

1. Basic Equations and Boundary Conditions. Equations for filtration and heat transport in moving rod bundles have been derived [3] via the traditional approach with evolution of a control volume. By using the simplifications introduced in [3], a system of equations for axisymmetric flow can be written in the form of equations for a boundary layer:

$$\varepsilon^{-1}\rho\left(u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial r}\right)=-\varepsilon\frac{dp}{dx}+R_x-\varepsilon^{-1}\rho u_1 G+\mu\left(\frac{\partial^2 u_1}{\partial r^2}+\frac{\partial u_1}{r\partial r}\right),$$

$$\frac{\partial(ru_1)}{\partial x}+\frac{\partial(rv_1)}{\partial r}=rG,$$

$$\varepsilon^{-1}\rho c_p\left(u_1\frac{\partial T_1}{\partial x}+v_1\frac{\partial T_1}{\partial r}\right)=\varepsilon^{-1}Q-\varepsilon^{-1}\rho c_p T_1 G+\lambda\left(\frac{\partial^2 T_1}{\partial r^2}+\frac{\partial T_1}{r\partial r}\right), \tag{1.1}$$

$$\varepsilon^{-1}\rho\left(u_1\frac{\partial c_1}{\partial x}+v_1\frac{\partial c_1}{\partial r}\right)=\varepsilon^{-1}K-\varepsilon^{-1}\rho c_1 G+\rho D\left(\frac{\partial^2 c_1}{\partial r^2}+\frac{\partial c_1}{r\partial r}\right).$$

Here u_1 and v_1 are the components of the filtration velocity corresponding to x and r; p is the pressure; T_1 is the temperature; c_1 is the concentration; ρ is the density; ε is the porosity; c_p is the heat capacity; μ is the dynamic viscosity; λ is the thermal conductivity; D is the diffusion coefficient; and the subscript 1 denotes parameters of filtration flow in the bundle.

The transport coefficients μ, λ, and D in Eqs. (1.1) should consider the dissipative processes in the hypothetical porous medium; therefore in the general case they differ from the coefficients of molecular viscosity, thermal conductivity, and diffusion. However, in many cases, authors use values which correspond to physical material parameters which are valid for rarefied structures when the twisting coefficient is close to unity [4, 5]. Equations (1.1) are written for heat and mass transport in a two-component medium. The presence

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of more components in the mixture should be accounted for by the corresponding transport equations for the other components. Here it should be noted that the transport coefficients, the density, and the heat capacity in the equations of motion and heat transport are averages and depend on the mass composition of the mixture and the physical parameters of the components.

The region of filtration flow in the bundle is joined to a homogeneous region. In order to describe motion and heat and mass transport in this region we use equations for the boundary layer. They can be obtained from (1.1) by setting $\varepsilon = 1$ and $R_x = Q = K = G = 0$. The subscript 2 is used for parameters in this region.

The interaction between regions is reflected by compatibility conditions at the boundary of the bundle ($r = R_b$):

$$u_2 = \varepsilon^{-1}u_1,\ v_2 = v_1,\ T_2 = T_1,\ c_2 = c_1,$$
$$\mu\frac{\partial u_2}{\partial r} = \mu\frac{\partial u_1}{\partial r},\quad \lambda\frac{\partial T_2}{\partial r} = \varepsilon\lambda\frac{\partial T_1}{\partial r},\quad D\frac{\partial c_2}{\partial r} = \varepsilon D\frac{\partial c_1}{\partial r}. \tag{1.2}$$

As boundary conditions we describe the conditions on the axis of symmetry and at infinity for a bundle in an open fluid and at $r = R_T$ for flow in a tube:

$$v_1 = 0,\quad \frac{\partial u_1}{\partial r} = \frac{\partial T_1}{\partial r} = \frac{\partial c_1}{\partial r} = 0 \qquad (r = 0),$$
$$u_2 = U_\infty,\quad T_2 = T_\infty,\quad c_2 = c_\infty \text{ or } \frac{\partial c_2}{\partial r} = 0\ (r \to \infty \text{ or } r = R_T). \tag{1.3}$$

Equations (1.1)-(1.3) determine the heat and mass transport in a moving fiber, bundle, if G, R_x, Q, and K are known.

2. Motion and Heat and Mass Transport in a Cell. The unknown G, R_x, Q, and K can be found via the cell method [6]. A modified cell model has been described [2, 3] which makes it possible to reflect the effect of the boundary conditions of the whole process on the distribution of the basic parameters in cells. By using this method, we will represent the flow in the neighborhood of a single fiber in the form of axisymmetric motion in a coaxial cylinder, whose internal radius coincides with the radius of the fiber (without loss of generality we will assume that the fiber is a long thin rod with radius R_{rod}), and whose outer radius has the form $R_\Delta = R_{rod}(1-\varepsilon)^{-1/2}$. In order to describe the transport processes in this cell, we make use of the boundary-layer equations, whose analytic solution will be sought via the method of successive approximations [7], where as the zero order approximation we use parameters which satisfy the equations

$$v\frac{\partial u}{\partial r} = -\frac{dp}{\rho dx} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r\partial r}\right),\quad \frac{\partial(rv)}{\partial r} = 0,$$
$$\rho c_p v\frac{\partial T}{\partial r} = \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{r\partial r}\right),\quad \rho v\frac{\partial c}{\partial r} = \rho D\left(\frac{\partial^2 c}{\partial r^2} + \frac{\partial c}{r\partial r}\right). \tag{2.1}$$

Equations (2.1) differ from the previous [2, 3] corresponding cell equations by the presence of convective transport components in the radial direction, which result from blowing gas on the surface of the fiber. The boundary conditions for this problem are

$$u = U_{rod}, v = v_{rod}, T = T_{rod}, c = c_{rod}\ (r = R_{rod}),$$
$$u = U_\Delta,\ T = T_\Delta,\ c = c_\Delta \qquad (r = R_\Delta), \tag{2.2}$$

where U_{rod} and T_{rod} are the velocity and temperature of the fiber; c_{rod} is the concentration on the surface of the fiber; and v_{rod} is the blowing gas velocity. The relationships at $r = R_{rod}$ correspond to the physical conditions of the problem, and the parameters U_Δ, T_Δ, and c_Δ are temporarily unknown quantities which are determined from the integral equations

$$\int_{R_{rod}}^{R_\Delta} rudr = \frac{1}{2}R_\Delta^2 u_1,\quad \int_{R_{rod}}^{R_\Delta} ruTdr = \frac{1}{2}R_\Delta^2 u_1 T_1,\quad \int_{R_{rod}}^{R_\Delta} rucdr = \frac{1}{2}R_\Delta^2 u_1 c_1. \tag{2.3}$$

The first expression indicates that the filtration velocity is averaged over the mass of the cell, the filtration gas temperature T_1 is averaged over the heat in the cell, and c_1 is averaged over the mass of the corresponding component.

By solving Eqs. (2.1) with the boundary conditions (2.2), we obtain

$$u = U_{\text{rod}} + (U_\Delta - U_{\text{rod}}) f_c(r, \alpha) + \frac{dp}{\mu dx} f_1(r, \alpha),$$
$$T = T_{\text{rod}} + (T_\Delta - T_{\text{rod}}) f_0(r, \alpha_T),$$
$$c = c_{\text{rod}} + (c_\Delta - c_{\text{rod}}) f_0(r, \alpha_c), \quad f_0(r, \alpha) = \frac{r^\alpha - R_{\text{rod}}^\alpha}{R_\Delta^\alpha - R_{\text{rod}}^\alpha}, \tag{2.4}$$
$$f_1(r, \alpha) = \frac{1}{2(2-\alpha)}\left[r^2 - R_{\text{rod}}^2 - (R_\Delta^2 - R_{\text{rod}}^2) f_0(r, \alpha)\right],$$
$$\alpha = \text{Re}_{\text{rod}} = v_{\text{rod}} R_{\text{rod}}/\nu, \ \alpha_T = \text{Re}_{\text{rod}} \text{Pr}, \ \alpha_c = \text{Re}_{\text{rod}} \text{Sc},$$
$$\text{Pr} = \mu c_p/\lambda, \ \text{Sc} = \nu/D.$$

The parameters U_Δ, T_Δ, and c_Δ are found with the aid of (2.3) from the expressions

$$U_\Delta = U_{\text{rod}} + S_1^{-1}(\varepsilon, \alpha)\left[u_1 - \varepsilon U_{\text{rod}} - \frac{dp}{\mu dx} S_2(\varepsilon, \alpha)\right],$$
$$T_\Delta = T_{\text{rod}} + B_T^{-1} u_1 (T_1 - T_{\text{rod}}),$$
$$c_\Delta = c_{\text{rod}} + B_c^{-1} u_1 (c_1 - c_{\text{rod}}).$$

$$B_T = U_{\text{rod}} S_1(\varepsilon, \alpha_T) + (u_1 - \varepsilon U_{\text{rod}}) \frac{S_3(\varepsilon, \alpha, \alpha_T)}{S_1(\varepsilon, \alpha)} + \frac{dp}{\mu dx}\left[S_4(\varepsilon, \alpha, \alpha_T) - \frac{S_2(\varepsilon, \alpha) S_3(\varepsilon, \alpha, \alpha_T)}{S_1(\varepsilon, \alpha)}\right], \tag{2.5}$$

$$B_c = U_{\text{rod}} S_1(\varepsilon, \alpha_c) + (u_1 - \varepsilon U_{\text{rod}}) \frac{S_3(\varepsilon, \alpha, \alpha_c)}{S_1(\varepsilon, \alpha)} + \frac{dp}{\mu dx}\left[S_4(\varepsilon, \alpha, \alpha_c) - \frac{S_2(\varepsilon, \alpha) S_3(\varepsilon, \alpha, \alpha_c)}{S_1(\varepsilon, \alpha)}\right].$$

The functions $S_i(\varepsilon, \alpha, \alpha_T)$ and $S_i(\varepsilon, \alpha, \alpha_c)$ are determined by

$$S_1(\varepsilon, \alpha) = \gamma(\varepsilon, \alpha) - \varepsilon\beta(\varepsilon, \alpha),$$
$$S_2(\varepsilon, \alpha) = \frac{\varepsilon R_\Delta^2}{2(2-\alpha)}[\varepsilon\beta(\varepsilon, \alpha) - \gamma(\varepsilon, \alpha) + \varepsilon/2],$$
$$S_3(\varepsilon, \alpha, \alpha_T) = \delta(\varepsilon, \alpha, \alpha_T) - \beta(\varepsilon, \alpha_T)\gamma(\varepsilon, \alpha) -$$
$$- \beta(\varepsilon, \alpha)\gamma(\varepsilon, \alpha_T) + \varepsilon\beta(\varepsilon, \alpha)\beta(\varepsilon, \alpha_T),$$
$$S_4(\varepsilon, \alpha, \alpha_T) = \frac{R_\Delta^2}{2(2-\alpha)}[\eta(\varepsilon, \alpha_T) - \varepsilon(1 - \varepsilon/2)\beta(\varepsilon, \alpha_T) -$$
$$- (1-\varepsilon) S_1(\varepsilon, \alpha_T) - \varepsilon S_3(\varepsilon, \alpha, \alpha_T)],$$
$$\beta(\varepsilon, \alpha) = \frac{(1-\varepsilon)^{\alpha/2}}{1-(1-\varepsilon)^{\alpha/2}}, \quad \gamma(\varepsilon, \alpha) = \frac{2}{\alpha+2}\frac{1-(1-\varepsilon)^{\alpha/2+1}}{1-(1-\varepsilon)^{\alpha/2}},$$
$$\delta(\varepsilon, \alpha, \alpha_T) = \frac{2}{\alpha+\alpha_T+2}\frac{1-(1-\varepsilon)^{(\alpha+\alpha_T+2)/2}}{[1-(1-\varepsilon)^{\alpha/2}]\left[1-(1-\varepsilon)^{\alpha_T/2}\right]},$$
$$\eta(\varepsilon, \alpha_T) = \frac{2}{\alpha_T+4}\frac{1-(1-\varepsilon)^{\alpha_T/2+2}}{1-(1-\varepsilon)^{\alpha_T/2}}.$$

Once we have the distributions of the velocity, temperature, and concentrations in the cell, we can find the unknown quantities R_x, Q, K, and G. But first we note that the velocity, the energy, and mass in the cell which surround the rod not only change from interaction with the fiber surface, but also depend on the blowing gas velocity. As a result, the volumetric sources of heat, mass, and friction in the bundle are determined in the form

$$R_x = \frac{2\pi R_{\text{rod}}}{\pi R_\Delta^2}(\rho U_{\text{rod}} v_{\text{rod}} - \tau_{\text{rod}}),$$
$$Q = \frac{2\pi R_{\text{rod}}}{\pi R_\Delta^2}(\rho c_p v_{\text{rod}} T_{\text{rod}} - q_{\text{rod}}), \ K = \frac{2\pi R_{\text{rod}}}{\pi R_\Delta^2}(\rho v_{\text{rod}} c_{\text{rod}} - g_{\text{rod}}) \tag{2.6}$$

where τ_{rod} is the friction stress on the fiber surface, and q_{rod} and g_{rod} are the thermal and diffusion fluxes from the fiber surface. By using (2.4) we find

$$\tau_{rod} = \mu \frac{\partial u}{\partial r}\Big|_{r=R_{rod}} = \mu (U_\Delta - U_{rod}) f_0'(R_{rod}, \alpha) + \frac{dp}{\mu dx} f_1'(R_{rod}\, \alpha), \tag{2.7}$$

$$q_{rod} = -\lambda (T_\Delta - T_{rod}) f_0'(R_{rod}, \alpha_T), \quad g_{rod} = -D (c_\Delta - c_{rod}) f_0(R_{rod}, \alpha_c).$$

The mass source G is computed from the velocity of the blowing gas to the surface or the removal of gas from the fiber surface

$$G = \frac{2\pi R_{rod}}{\pi R_\Delta^2} v_{rod}.$$

Equations (2.6) and (2.7) complete the formulation of the above problem of heat and mass transport in the bundle of moving fibers.

3. Calculated Results. The parameter distributions (2.4) show that gas evolution on the fiber surfaces substantially changes the form of the solutions in the cell. The distributions of the basic parameters in this case are given in the form of power functions (as opposed to logarithmic functions) of the transverse coordinates, which functions were obtained for flow without blowing gas. There is an asymptotic transition from the power functions to logarithmic functions as $\alpha \to 0$; as a result Eq. (2.4) can be considered to be a generalization of previous results. The effect of blowing gas can be more clearly shown by comparing the friction stress on the rod with and without blowing gas. Table 1 shows the ratio $\zeta = \tau_{rod}/\tau_0$, where τ_0 is the friction without blowing gas, as a function of the blowing parameter $\alpha = Re_{rod}$ for various values of the porosity ε. The friction stresses are compared for identical filtration velocities in the cell for the simplest case – forced convection in an open fiber bundle ($dp/dx = 0$). From the table it can be seen that τ_{rod} decreases as the blowing increases, and friction increases for gas evolution ($v_{rod} < 0$). This behavior of the curves corresponds qualitatively to the way the friction varies for blowing gas or gas evolution in the boundary layer of a single cylinder. The condition $\tau_{rod}/\tau_0 \to 1$ is fulfilled as $\alpha \to 0$. The ratios q_{rod}/q_0 and g_{rod}/g_0 for the heat and mass flows behave in an analogous manner.

The change of local friction parameters in the cell caused by gas evolution makes possible the assumption that the evolution has a substantial effect on the heat and mass transport of the whole bundle. In actual industrial processes for working or forming fibers, the evolution of liquid or gas is a result of chemical reactions in the volume of the fiber or else is determined by the rate of phase transitions at the fiber surface. Today explicit functions for many kinetic processes either are unavailable or else they are rather complicated and must be examined separately for each process [8]. In this regard, here we examine problems in which the magnitude of the blowing gas is either specified or established from comparatively simple functions of the coupling parameters at the surface of the thread.

We will assume that the bundle has the following characteristic parameters: radius R_b = 50 mm; fiber radius R_{rod} = 0.125 mm; number of fibers N = 140; velocity U_{rod} = 0.5 m/sec; fiber temperature T_{rod} = 270°C; temperature of the fluid or the tube wall $T_\infty = T_w$ = 20°C; and tube radius R_T = 70 mm. In the first group of problems, the blowing gas velocity v_{rod} is taken constant and identical for all fibers in the bundle. The calculations were done for the following set of Reynolds numbers for the blowing gas: α = 0, 0.005, 0.05, and 0.5. Numerical solutions of Eqs. (1.1)-(1.3), (2.6), and (2.7) were obtained by the method in [9].

Figures 1 and 2 show the distributions of the filtration velocities and temperatures of the gas on the axis (curve s) and of the fiber surface (curve p), which moves in an immobile inorganic fluid (curves 1-4 for α = 0, 0.005, 0.05, and 0.5). It can be seen that the gas velocities and temperatures are much lower on the surface of the fiber bundle than in the center. This is caused by the development of flow within the bundle, which decreases the thickness of the boundary layer and limits the zone for transport of gas from the external fluid to the space between the rods. As the flow develops, the transport flows decrease, and the velocities and temperatures of the gas start to increase. They tend monotonically to their limiting values of U_{rod} and T_{rod}. Here it can be seen that the rate of change of the velocities and the temperatures depends strongly on the blowing parameter α. For α = 0.5, even for x/R_b = 20, the filtration velocities and temperatures almost coincide with U_{rod} and T_{rod}, and have a stabilized homogeneous profile throughout the volume of the bundle.

Distributions of the velocities and temperatures along the length of the channel are shown in Figs. 3 and 4. Here, as in the previous case, we show curves which characterize

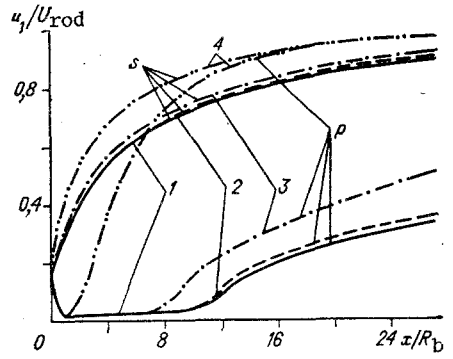

Fig. 1

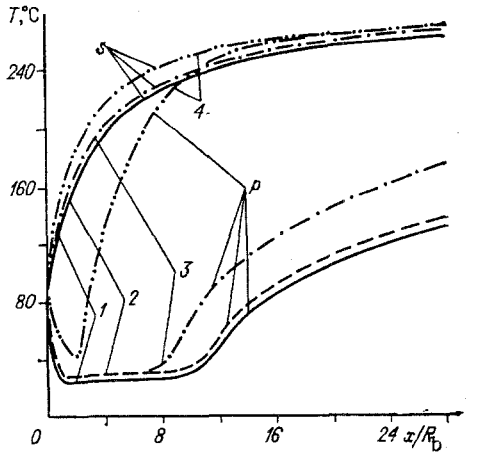

Fig. 2

TABLE 1

α	ε			
	0,7	0,8	0,9	0,95
	ζ			
—0,5	1,11	1,16	1,24	1,32
—0,3	1,07	1,09	1,14	1,19
—0,1	1,02	1,03	1,05	1,06
0	1	1	1	1
0,1	0,98	0,97	0,96	0,94
0,3	0,94	0,91	0,87	0,83
0,5	0,89	0,86	0,79	0,73

parameter changes on the axis and at the boundary of the bundle (the notation coincides with that in Figs. 1 and 2). The initial distributions for the filtration velocity were assumed homogeneous and equal to the velocity of the fiber motion. This would make it possible to exclude the effect of the dynamic initial section and to show the effect of blowing gas on hydrodynamics and heat transport in the bundle. In Fig. 3 it can be seen that the velocities change linearly in the channel, and the rate of change is determined by α. Here the velocity is somewhat higher at the bundle boundary on the axis. This behavior of the flow is caused by the transport of gas from within the bundle to the region of homogeneous flow for $R_b < r < R_T$ and by the formation of the maximum longitudinal velocity within this region. The gas temperature increases monotonically with x and approaches T_{rod}; however, as compared with the heat transport in an open fluid, the temperature change in the channel occurs much slower in this case – and reflects the effect of the wall.

In these examples, we obtained solutions of problems of blowing a homogeneous gas. For investigations of mass transport, there is interest in case of blowing a foreign gas, which occur, for example, for liquid evaporation on the surface of the thread. In the following group of problems we examine an isothermal system of rods, on whose surfaces the concentration of the evolved component c_{rod} is specified as a constant. As follows from the boundary conditions, a hydrodynamic flow with a blowing velocity [10]

$$v_{rod} = -\frac{D}{1-c_{rod}}\frac{\partial c}{\partial r}\Big|_{r=R_{rod}}. \tag{3.1}$$

is formed for mass flows on the separation surface, which is impermeable to the fluid. The relationshp between C_{rod}, v_{rod}, and $\partial c/\partial r|_{R_{rod}}$ can be obtained from the analytical solutions of (2.4) and (2.5), after the problem has been solved completely.

The (solid) curves in Fig. 5 (for an open bundle) and Fig. 6 (for flow in a tube) show the change the concentrations of the evolved component on the axis and on the surface of the bundle. The calculations are done for $c_{rod} = 0.4$ and 0.8 (curves 1 and 2); here we took $c_\infty = 0.02$ and Sc = 3. The geometric dimensions and the fiber velocities in these problems were taken the same as in the previous cases. Figures 5 and 6 also show the distribution of the blowing parameter α (dashed curves) which were computed by using Eq. (3.1). These cal-

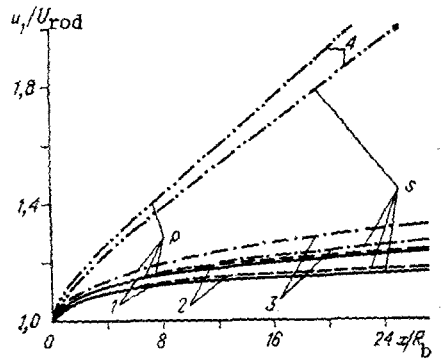

Fig. 3

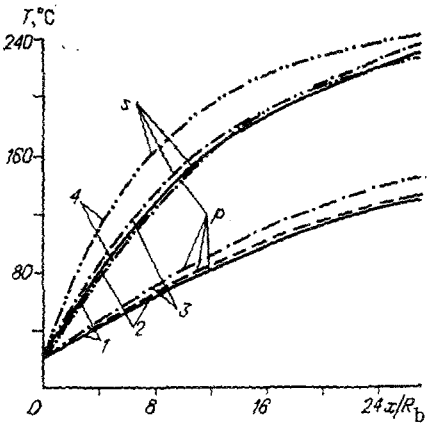

Fig. 4

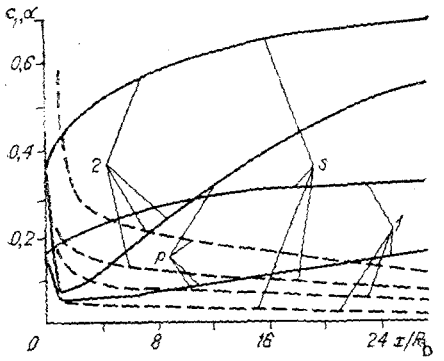

Fig. 5

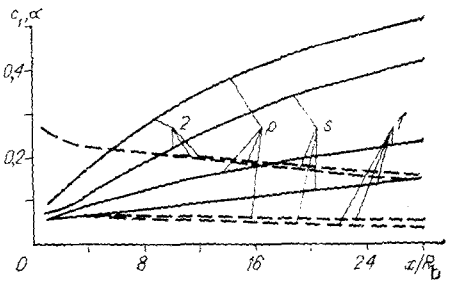

Fig. 6

culations show that the foreign gas concentration in the center part of the bundle rises much faster than at the periphery (it reflects the effect of fluid transport from the outer fluid to the surface layers of the bundle). Increasing c_{rod} increases the mass transport, and relatively rapid evaporation is observed on the surface fibers in the initial section ($\alpha \sim 0.5$ for $c_{rod} = 0.8$). As the flow and the formation of an outer boundary layer develop, the gradients $\partial c/\partial r|_{R_{rod}}$ decrease and lower the gas evolution. Mass transport in a channel is characterized by a smoother change in concentration. Moreover, due to the more uniform distribution of the filtration velocity through the bundle thickness, the difference between $c_{r=R_b}$ and $c_{r=0}$ and also between $\alpha_{r=R_b}$ and $\alpha_{r=0}$ decreases substantially. At the same time higher values of the gas evolution rates are observed throughout the bundle. These features lead to the conclusion that cooling or processing fibers in a channel is better than in an open fluid.

In conclusion, we note that these numerical solutions allow the conclusion that the proposed flow model is applicable for calculations of heat and mass transport processes in bundles of moving fibers with gas evolution. The asymptotic behavior of the solutions for $\alpha \to 0$ and their transition to known tested functions can serve as a qualitative evaluation of the reliability of our results. Obtaining quantitative estimates of the accuracy of this method is currently very difficult due to the absence of experimental investigations. In spite of this, the substantial effect of gas evolution on the distribution of the basic parameters, especially for heat and mass transport in an open fluid, demonstrates the urgency of further theoretical study of these processes.

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DYNAMIC DISINTEGRATION AND EXPANSION OF A LIQUID VOLUME

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The process of dispersing (disintegrating) a liquid volume and forming a droplet–air cloud during an explosion can be divided into the following [three] stages. The first – the propagation of an explosive wave which arises when the charge detonates – is essentially determined by an explosion in an unbounded liquid, and has been studied theoretically and experimentally [1-5].

The second stage starts when the shock wave propagating through the liquid reaches a free surface, reflects, and produces a rarefaction. The tensile stress which arises behind the rarefaction front leads to the intense production of vapor–gas bubbles – the cavitation phenomenon [3, 6-8]. The solution to the problem of bubble cavitation has been examined in [9-14]. In particular, the explosive loading of a cylindrical liquid layer has been studied in [15-18]. Depending on the magnitude of the specific energy release it has been established that either cavitation disintegration or hydromechanical perturbations (of the Rayleigh–Taylor type) can occur on the inner and outer surfaces of the liquid volume. A break-away disintegration, which has been observed experimentally [3, 19] is also possible. The second stage ends when the volume concentration of bubbles in the liquid reaches a critical value, at which an inversion process takes place: the bubble-filled liquid transforms to a droplet stage.

The third stage starts with the formation of a droplet–vapor mixture; as it moves it is blown by the reverse flow of the surrounding air on the outer boundary and by the detonation products on the inner boundary. The expansion of the initial finely dispersed particles or droplets and estimates of the dimension of the resultant droplet–air cloud is discussed in [20] and [21].

Here we study the problem of the explosive dispersion of a liquid volume, the subsequent expansion of the resultant droplet–air cloud [22] in spherical and cylindrical geometry, and propose an approximate numerical model. The problem is examined for large-scale phenomena when the relaxation time of the tensile stresses in the rarefaction wave are small compared to a characteristic hydrodynamic time scale.

1. We study a solid chemical explosive charge with an initial density ρ_{ex} and radius R_{0c} which is surrounded by a liquid layer with a radius R_{0k}. The initial liquid pressure is p_{01} with density ρ_{01}. The liquid is surrounded by infinite air with an initial pressure p_{02} and density ρ_{02}. In the spherical case it is assumed that at time $t = 0$ an energy W is instantaneously released in a volume $v_i = 4/3\pi R_{0c}^3$; the intial pressure p_i of the detonation products is found from [5]

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